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Measurements of the Dalitz Plot Parameters for $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ Decays

Abstract

The g, h, and k Dalitz plot parameters, which are coefficients in a series expansion of the squared module of the matrix element $|M(u,v)|^2 \propto 1+gu+hu^2+kv^2$ (u, v are invariant variables), have been measured for $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays using 35 GeV/c hadron beams at the IHEP (Protvino) accelerator. Dependences of parameters and fit quality on the $\pi^0\pi^0$ mass cut were investigated. The results point to the important role of $\pi^+\pi^- \to \pi^0\pi^0$ charge exchange scattering near the $\pi^0\pi^0$ mass threshold. The comparison of our data with previous measurements is presented.

1 Introduction

The Dalitz plot parameters for $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays measured in the three most precise experiments with K^- [1,2] and K^+ [3] beams differ by 2 to 5 standard deviations [4]. For example, the difference of the Dalitz plot slopes g obtained in [2] and [3] is equal to 0.109 ± 0.021. As shown by our data [5] this result cannot be explained by CP violation and is most probably due to underestimation of systematic uncertainties. In this paper we present new results on the Dalitz plot parameters based on the analysis of 493k events of $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays collected with the TNF-IHEP facility [5,6].

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2 Experimental Setup

Studies of charged kaon decays have been performed using 35 GeV/c positive and negative hadron beams at the IHEP accelerator. The beam intensity was monitored by four scintillation counters. Its typical value was $4 \cdot 10^6$ per 1.7 second spill. Three threshold and two differential Cherenkov counters were used to select kaons with a background of less than 1%. The products of kaon decays originating in the 58.5 m long vacuum pipe were detected by wide aperture scintillation hodoscopes and the total absorption electromagnetic calorimeter GEPARD consisting of 1968 lead-scintillator cells. The π^0 mass resolution was 12.3 MeV/c^2. The calorimeter was divided into 16 trigger elements. An anticoincidence beam counter was placed downstream of the vacuum pipe. The first level trigger T1 was formed according to the following logic formula:

$$T1 = S1 \cdot S2 \cdot S3 \cdot S4 \cdot (D1 + D2) \cdot \overline{C1} \cdot \overline{C2} \cdot \overline{C3} \cdot \overline{AC},$$

where Si, Di, Ci, and AC are logical signals from the beam, differential, threshold, and anticoincidence counters respectively. The Level 2 trigger required more than 0.8 GeV energy deposition in at least three trigger elements of the GEPARD. The details of the setup and measurement procedure can be found elsewhere [5].

3 $K^\pm \to \pi^\pm \pi^0 \pi^0$ event selection

The following criteria were used to select $K^\pm \to \pi^\pm \pi^0 \pi^0$ events [5]:

- one to three secondary tracks are reconstructed;
- the probability of the decay vertex fit is more than 5%;
- the decay vertex is inside the fiducial length of the decay pipe;
- the number of clusters with energy above 1 GeV in the calorimeter and the number of tracks in the hodoscopes correspond to the $K^\pm \to \pi^\pm \pi^0 \pi^0$ decay;
- charged pion energy exceeds 8 GeV;
- the χ^2 probability $P(\chi^2)$ of the 6C kinematic fit is more than 0.1 (all possible photon combinations are considered and the best is selected);
- event passes software Level 2 trigger.

The experimental setup was simulated using the Monte Carlo (MC) method with the GEANT 3.21 code. The setup geometry was described in detail and the data obtained in the experiment were taken into account. Among these data there were calibration coefficients for each channel of the calorimeter, the dependence of the hodoscope efficiency on the particle coordinates and correlations between kaon's spatial and angular coordinates and its momentum.

Table 1
Fit results with higher-order terms in (1)

g	0.6259 ± 0.0043	0.6151 ± 0.0051	0.6284 ± 0.0048	0.6129 ± 0.0063
h	0.0551 ± 0.0044	0.0782 ± 0.0073	0.0556 ± 0.0044	0.0795 ± 0.0077
k	0.0082 ± 0.0011	0.0080 ± 0.0011	0.0070 ± 0.0015	0.0087 ± 0.0016
l	—	0.0273 ± 0.0069	—	0.0292 ± 0.0076
m	—	—	-0.0027 ± 0.0024	0.0016 ± 0.0027
χ^2	506.1	490.2	504.8	489.8
χ^2/ndf	1.18	1.14	1.17	1.14
$P(\chi^2)$	$6.6 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$	$7.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$

The final data sample includes 493K completely reconstructed $K^\pm \to \pi^\pm\pi^0\pi^0$ events. The MC statistics is about four times higher. The background level estimated from the MC simulations is less than 0.25% and is mainly due to $K^\pm \to \pi^\pm\pi^0$ decays. It is shown in our paper [5] that the event distributions in the Dalitz plots for K^+ and K^- decays are identical. Taking this into account we used combined statistics for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays to estimate the Dalitz plot parameters.

4 Results

The following parametrization of the squared module of the matrix element for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays was used in the data analysis [4]:

$$|M(u,v)|^2 \propto 1 + gu + hu^2 + kv^2 \tag{1}$$

Due to the finite setup resolution on the u and v variables [5] the 'measured' u', v' values can differ from the true u, v for both experimental and MC events. To take this into account the Dalitz plot parameters were estimated by minimizing the following functional form:

$$\chi^2(g,h,k) = \sum_i^{Nbin} \frac{(n_i - C \cdot m_i))^2}{\sigma_i^2},$$

where n_i is the number of events in the i-th experimental Dalitz plot bin, $m_i \equiv m_i(g,h,k) = \sum_j w_{ij}$ $(w_{ij} = 1 + g \cdot u_j + h \cdot u_j^2 + k \cdot v_j^2)$ is a sum of the weighted MC events in the i-th Dalitz plot bin, $C = \sum n_i / \sum m_i$ is a

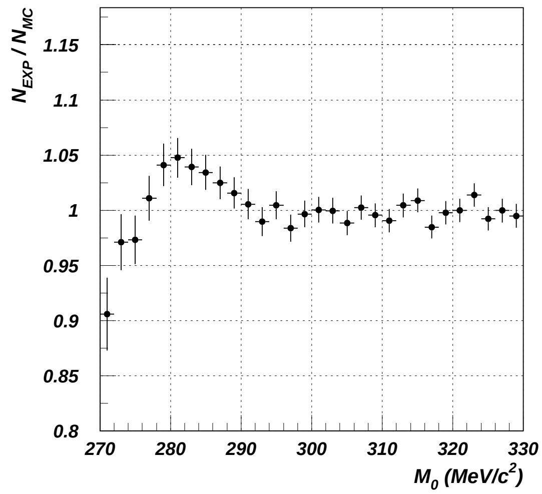

Fig. 1. Ratio of experimental to MC events vs $\pi^0\pi^0$ invariant mass M_0.

normalization factor and $\sigma_i^2 = n_i + C^2 \cdot \sum\limits_j w_{ij}^2$ takes into account limited MC statistics. The following values of the g, h, k parameters and elements of the correlation matrix were obtained:

$$\begin{cases} g = 0.6259 \pm 0.0043, \\ h = 0.0551 \pm 0.0044, \\ k = 0.0082 \pm 0.0011, \end{cases} \qquad \begin{pmatrix} 1.00 & 0.90 & 0.41 \\ & 1.00 & 0.33 \\ & & 1.00 \end{pmatrix}. \tag{2}$$

The errors quoted are statistical only, χ^2/ndf is $506/430 = 1.18$ and $P(\chi^2) = 0.0066$. The low significance of the fit is primarily due to a difference between the experimental data and the MC simulations based on equation (1) in the threshold region of the $\pi^0\pi^0$ invariant mass M_0 (Fig. 1). This discrepancy can not be avoided by the addition of higher order terms $l \cdot u^3$ and/or $m \cdot uv^2$ in the $|M(u,v)|^2$ expansion (see Table 1) and might be due to nonanalitical terms in the matrix element connected to $\pi^+\pi^- \to \pi^0\pi^0$ rescattering [7]. This effect was recently considered in detail by Cabibbo [8] and Cabibbo, Isidory [9]. It plays an important role in the region of $M_0 \sim 2m_{\pi^0}$ and its contribution can be suppressed by introducing a complementary criterion of $M_0 > M_T$. It appeared that the fit with $M_T \geq 290\ MeV/c^2$ results in stable values of the Dalitz plot parameters independent from the M_0 cut and a satisfactory fit significance. Introduction of the higher order terms in Eq.(1) does not change the g, h, k parameters and χ^2 value in this case. The fit with $M_T = 290\ MeV/c^2$ gave

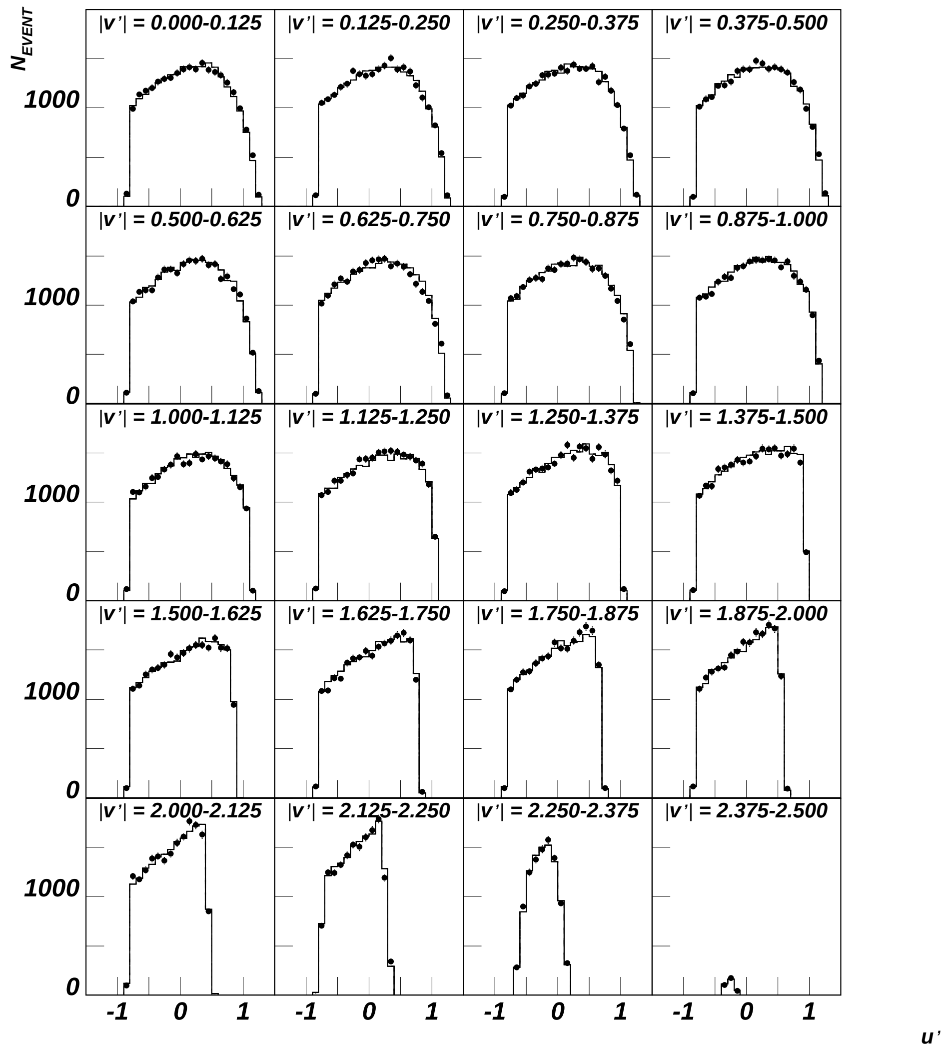

Fig. 2. The u' distribution of events in different intervals of $|v'|$ (histogram – simulation, circles – experiment).

the following results:

$$\left\{ \begin{aligned} g &= 0.6339 \pm 0.0046, \\ h &= 0.0593 \pm 0.0088, \\ k &= 0.0083 \pm 0.0013, \end{aligned} \right. \qquad \begin{pmatrix} 1.00 & 0.52 & 0.43 \\ & 1.00 & 0.16 \\ & & 1.00 \end{pmatrix}, \tag{3}$$

and $\chi^2/ndf = 1.04$, $P(\chi^2) = 0.3$.

Comparison of the matrices in (2) and (3) shows that the correlations between g, h, and h, k are much weaker (significantly lower) if the M_T cut is applied. Figs. 2-4 confirm the good agreement between experimental and MC data with $M_T = 290\ MeV/c^2$.

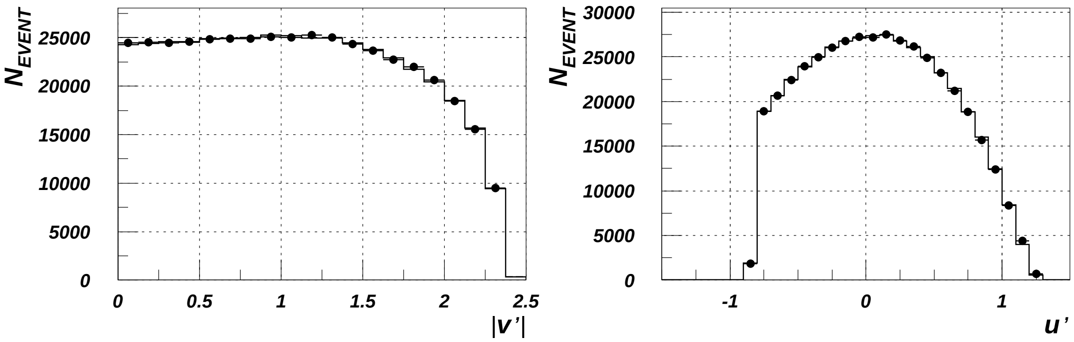

Fig. 3. Event distributions projected on the u' and $|v'|$ axes (histogram - simulation, circles - experiment).

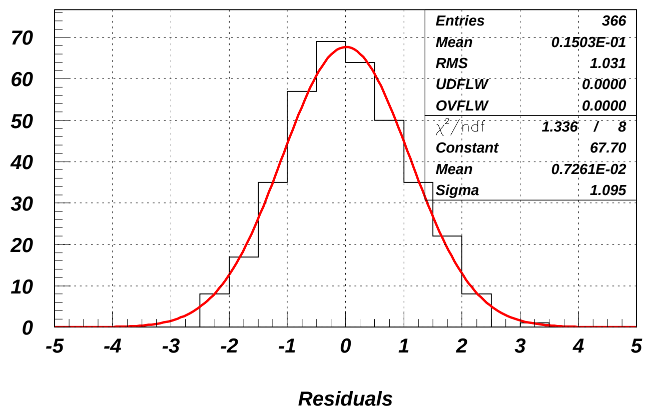

Fig. 4. Normalized residuals of the fit $(n_i - m_i)/\sigma_i$.

To estimate the systematic uncertainties of the Dalitz plot parameters we checked the stability of the results against variation of the cuts in the event selection criteria. The parameters appeared to be most sensitive to the change of the minimum gamma energies from 1 to 2 GeV ($\Delta g = -0.0057$, $\Delta h = -0.0047$, $\Delta k = -0.0006$) and of the minimum charged pion momentum from 1 to 8 GeV/c ($\Delta g = 0.0048$, $\Delta h = 0.0051$ and $\Delta k = 0.0011$). The change in the bin size by factors of 2 and 0.5 and exclusion of the bins at the Dalitz plot boundary from the fit gives $\Delta g = 0.0012$, $\Delta h = 0.0045$, and $\Delta k = 0.0004$. Uncertainties in the kaon momentum, beam profile and angular spread, as well as GEPARD calibration coefficients have no influence on the parameters. The background contribution to systematic errors turned out to be negligible. Finally, our estimations of the systematic uncertainties are the following: $\delta g = 0.0093$, $\delta h = 0.0086$, $\delta k = 0.0014$. They do not include the errors connected with g, h, k variations due to introducing the M_0 cut and the higher order terms in the expansion (1).

Fig. 5 shows our results (2) together with previous measurements of the Dalitz plot parameters [1,2,3,4,10,11,12,13,14]. The error bars include both systematic and statistical uncertainties.

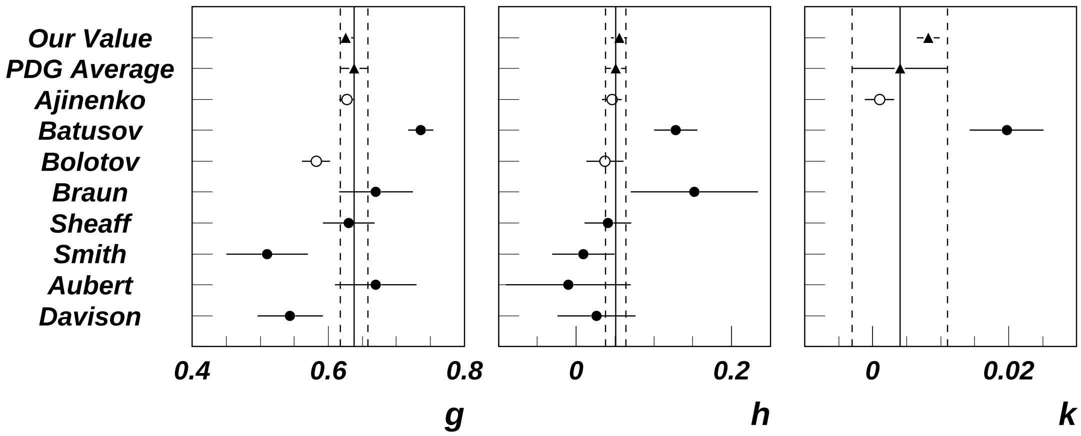

Fig. 5. The Dalitz plot parameters g, h and k for the $K^+ \to \pi^+\pi^0\pi^0$ (solid circles), $K^- \to \pi^-\pi^0\pi^0$ (open circles) and $K^\pm \to \pi^\pm\pi^0\pi^0$ (triangles) decays. Vertical solid and dotted lines show the average values and their errors as calculated by the PDG [4].

5 Conclusions

The new data on the Dalitz plot parameters for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays based on the analysis of $\sim 0.5M$ events collected with the TNF-IHEP facility are presented. The following results were obtained without a cut on the $\pi^0\pi^0$ invariant mass: $g = 0.6259 \pm 0.0043\,(stat) \pm 0.0093\,(syst)$, $h = 0.0551 \pm 0.0044\,(stat) \pm 0.0086\,(syst)$, $k = 0.0082 \pm 0.0011\,(stat) \pm 0.0014\,(syst)$. The g and h values are in good agreement with those of Ajinenko *et al* [2]. We observe a deviation of the k value from zero ~ 4.5 standard deviations while Ajinenko *et al* reported $k = 0.001 \pm 0.002$. We investigated the dependence of the Dalitz plot parameters and the fit quality on the $\pi^0\pi^0$ invariant mass cut M_T. It turned out that the fit significance becomes rather high if a cut of $M_T \geq 290\ MeV/c^2$ is applied. With this cut the addition of higher order terms in expansion (1) does not influence the g, h, and k parameters and the value of χ^2/ndf. These results may be considered as evidence of the important contribution of $\pi^+\pi^- \to \pi^0\pi^0$ rescattering [8,9] to the matrix element of the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay in the threshold region of the $\pi^0\pi^0$ invariant mass.

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References

[1] V.N. Bolotov *et al.*, Sov. J. Nucl. Phys., 44, 73 (1986).

[2] I.V. Ajinenko *et al.*, Phys.Lett. B567, 159 (2003).

[3] V.Y.Batusov *et al.*, Nucl. Phys. B516, 3 (1998).

[4] S. Eidelman *et al.*, Phys. Lett. B592, 1 (2004).

[5] G.A. Akopdzhanov *et al.*, Eur. Phys. J. C 40, 343 (2005).

[6] V.V. Ammosov *et al.*, Preprint IHEP 98-2, Protvino, 1998

[7] V.V. Anisovich, L.G. Dakhno, A.K. Likhoded, Sov. J. Nucl. Phys., 8, 91 (1968).

[8] N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004).

[9] N. Cabibbo, G. Isidori, JHEP 503, 21 (2005).

[10] D. Davison *et al.*, Phys. Rev., 180, 1333 (1969).

[11] B. Aubert *et al.*, Nuovo Cim., 12A, 509 (1972).

[12] K.M. Smith *et al.*, Nucl. Phys., B91, 45 (1975).

[13] M. Sheaff, Phys. Rev., D12, 2570 (1975).

[14] H. Braun *et al.*, Lett. Nuovo Cim., 17, 521 (1976).